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1. Write down the definition of a *valid deduction*

2. Use the **definition** of “validity” to prove that the following deduction is valid.

$$\begin{array}{l} P \rightarrow Q \\ Q \rightarrow R \\ \hline P \rightarrow R \end{array}$$

This is another “basic valid deduction,” called *Transitivity*. You should **memorize it!**

3. There is one last “basic valid deduction.”

$$\begin{array}{l} P \vee Q \\ P \rightarrow R \\ Q \rightarrow S \\ \hline R \vee S \end{array}$$

This argument is called *Dilemma*. You should **memorize** this one too!

Write down an informal argument (in words) explaining why this is true.

(**Hint:** in a “dilemma,” you are in one of two cases. Look at what happens in each case.)

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Here are some examples of arguments. carefully read the argument.

Then, **on a separate piece of paper**

1. translate it into symbolic logic, stating which letter you use for each *basic* statements, and
2. determine if the argument is valid or not.

Be sure to justify each step with a valid equivalence or argument. Good Practice!

1. If we go north then we ski.  
We go north.  
-----  
We ski.
2. If I do not sleep then I do not study.  
I do not sleep.  
-----  
I do not study.
3. If we are late we will miss lunch.  
We miss lunch.  
-----  
We are late.
4. If we are late we will miss lunch.  
We didn't miss lunch.  
-----  
We are not late.
5. Either you pay your bill or you don't graduate.  
You graduate.  
-----  
You paid your bill.
6. If she studies a lot she will do well.  
She will not be happy if she does not do well.  
-----  
If she studies a lot she will be happy.
7. If the temperature reaches  $-22$  F, gypsy moth eggs will freeze.  
If we have gypsy moths in the spring, their eggs did not freeze.  
The temperature reached  $-22$  F.  
-----  
There will be no gypsy moths this spring.
8. If I do my job I will be rewarded.  
Either I do my job or I go hiking.  
If I go hiking then I will feel good.  
-----  
Either I will feel good or I will be rewarded.
9. If we pack a lunch then we must buy bread.  
If we don't pack a lunch then we don't go on a picnic.  
If we buy bread then we must go to the store.  
-----  
We must go to the store.
10. Hot, humid weather favors thunderstorms.  
Hot, humid weather favors bacterial growth.  
Bacterial growth causes milk to sour.  
-----  
Thunderstorms cause milk to sour.

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11. Either I don't go to college or I get a good job.  
Either I make a lot of money or I don't get a good job.  
If I do not work hard then I do not make a lot of money.  
I go to college.  
-----  
I do not work hard.
12. If my car breaks down then I will not go to New York.  
If I get \$500 then I will go to .  
My car breaks down.  
-----  
I didn't get \$500.
13. All koalas are shy.  
-----  
All shy animals are friendly.  
-----  
All koalas are friendly.
14. Either my uncle is an art critic or muffins are unhealthy.  
If aliens write poetry, then muffins are unhealthy and my uncle is an art critic.  
-----  
But it is assuredly not true both that muffins are unhealthy and aliens do not write poetry.  
-----  
Hence, muffins are unhealthy and my uncle is an art critic.
15. If the plants are healthy then the trees are tall.  
The soil is not rich or the days are short.  
-----  
If the days are not short then the sun shines.  
-----  
The trees are tall and the soil is rich.
16. Either the sun is shining or the road is not rough.  
If the quality is good then product is not plastic.  
Either the product is plastic or the sun is not shining.  
The road is rough.  
-----  
The quality is not good.
17. If I am in New England then it is not the case that both it is winter and there is no snow.  
Either it is not the case that both it is winter and there is no snow or if I am not inside then I am cold.  
I am inside and I am in New England.  
-----  
I am not cold.

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*Some solutions:* Look at each argument, use letters to represent the statements, then determine if the argument is valid or not.

1) Let N represent “we go north” and S represent “we ski”.

Then	If we go north then we ski.	becomes	$N \rightarrow S$
	We go north.		$N$
	Wek Ski.		$S$

which is exactly the form of MP, so it *is* a valid argument.

Similarly,

2) Let S represent “I sleep” and D represent “I study”.

Then	If I do not sleep then I do not study.	becomes	$\neg S \rightarrow \neg D$
	I do not sleep.		$\neg S$
	I do not study.		$\neg D$

which is again exactly the form of MP, since  $\neg S$  is one premise and  $\neg S \rightarrow \neg D$  is the other premise, so by MP we conclude  $\neg D$ . Thus, this is a valid argument.

3) Not valid: Let L be F and M be T.

4) Valid by MT

5) Valid by DS

6) Consider the argument

$S \rightarrow W$	.
$\neg W \rightarrow \neg H$	
$S \rightarrow H$	

Note that witnessing that this argument is invalid is the same as making the implication

$$((S \rightarrow W) \wedge (\neg W \rightarrow \neg H)) \rightarrow (S \rightarrow H)$$

false. To do this, we try to make the hypotheses true while making the conclusion false. The only way that the conclusion,  $S \rightarrow H$ , could be false would be if  $S$  were true and  $H$  were false.

Now to make the premise,  $S \rightarrow W$ , true, since  $S$  must be true (to keep the conclusion false) we make  $W$  true. Note that  $\neg W \rightarrow \neg H$  is equivalent to  $H \rightarrow W$  by contrapositive. So we can make  $H \rightarrow W$  true by making  $H$  false.

In other words, this truth assignment  $\frac{S}{T} \mid \frac{H}{F} \mid \frac{W}{T}$  makes the conclusion false while keeping the premises true, thus demonstrating that the argument is not valid.

7) Note that it is probably not a good idea to represent statements by T or F. So, “the temperature reaches  $-22$  F” should be represented by TE or TP or TEMP but not by T, because that would create a confusion with T for true.

$TEMP \rightarrow EF$
$G \rightarrow \neg EF$
$TEMP$
$\neg G$

Now,  $TEMP$  together with  $TEMP \rightarrow EF$  gives  $EF$  by Modus Ponens (MP).

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Also,  $G \rightarrow \neg EF \equiv EF \rightarrow \neg G$  by contrapositive.

Finally,  $EF$  (obtained above) and  $EF \rightarrow \neg G$  together give  $\neg G$  by MP.

Thus the argument is valid.

8) Valid by dilemma

9) Not valid. We attempt to show this argument is invalid. First, to make the conclusion false we let  $S$  be  $F$ . Then, to make the premise,  $B \rightarrow S$ , true, we must label  $B$  as  $F$ . Working, in this way, since we have  $L \rightarrow B$ , then  $L$  must be  $F$  to make the whole statement true. Finally, since  $L$  must be false then  $P$  must also be false to make  $\neg L \rightarrow \neg P$  true. Now we have found a truth assignment which makes the conclusion false and all the premises true, so the argument is not valid.

10) This argument can be translated as

$$\begin{array}{l} H \rightarrow S \\ H \rightarrow B \\ B \rightarrow M \\ \hline S \rightarrow M \end{array}$$

We can obtain  $H \rightarrow M$  by transitivity, but not  $S \rightarrow M$ . So try to show it is not valid by making  $S \rightarrow M$  false and all the hypotheses true. How could  $S \rightarrow M$  be false?  $S$  would have to be true and  $M$  would have to be false. Working on trying to make the premise,  $B \rightarrow M$ , true, since  $M$  is false, we must make  $B$  false.  $B$  false forces  $H$  to be false. This truth assignment makes all the premises true and the conclusion false, so the argument is not valid.

11) Not valid. Use C, GJ, M, WH all true.

12)

$$\begin{array}{l} C \rightarrow \neg NY \\ FIVE \rightarrow NY \\ C \\ \hline \neg FIVE \end{array}$$

Notice that  $FIVE \rightarrow NY \equiv \neg NY \rightarrow \neg FIVE$  by contrapositive.

So,  $C$  and  $C \rightarrow \neg NY$  give us  $\neg NY$ .

$\neg NY$  and  $\neg NY \rightarrow \neg FIVE$  give us  $\neg FIVE$ .

Thus the argument is valid.

Note that there are several ways to show that this argument is valid. Here are two:

1. Contrapositive plus 2 applications of MP MP plus MT
2. Contrapositive plus transitivity plus MP

(13)

$$\begin{array}{l} \text{If } x \text{ is a koala then } x \text{ is shy.} \\ \text{If } x \text{ is shy then } x \text{ is friendly} \\ \hline \text{If } x \text{ is a koala then } x \text{ is friendly.} \end{array}$$

Valid by transitivity.

14)

$$\begin{array}{l} \text{Either my uncle is an art critic or muffins are unhealthy.} \\ \text{If aliens write poetry, then muffins are unhealthy and my uncle is an art critic.} \\ \text{But it is assuredly not true both that muffins are unhealthy and aliens do not write poetry.} \\ \hline \text{Hence, muffins are unhealthy and my uncle is an art critic.} \end{array}$$

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$$\begin{array}{l}
 \text{This is equivalent to } U \vee \neg M \\
 A \rightarrow (\neg M \wedge U) \\
 \neg(\neg M \wedge \neg A) \\
 \hline
 \neg M \wedge U
 \end{array}$$

There are three ways that the conclusion could be false: (i) both  $\neg M$  and  $U$  could be false (so  $M$  is True and  $U$  is False), or (ii)  $\neg M$  could be False and  $U$  could be True (so  $M$  is True and  $U$  is True), or (iii)  $\neg M$  could be True and  $U$  could be False (so  $M$  is False and  $U$  is False).

Now let's be careful here. What would we need to do to show that the argument is valid? We would have to make sure that it was IMPOSSIBLE to make all the premises true while the conclusion was false. To show that the argument is invalid we would only need to find one truth assignment that made all the premises true while the conclusion was false. Suppose we tried case (iii) where  $M$  False and  $U$  False (remember this makes the conclusion false). Since  $M$  is False, then  $\neg M$  is True, so the first premise is true. Since  $M$  is False, then  $\neg M$  is True and  $U$  is False so the conclusion of the second premise is False. Thus the only way the second premise could be true is if  $A$  is False. But then the third premise has  $\neg M$  True and  $\neg A$  True, so  $(\neg M \wedge \neg A)$  is True and thus,  $\neg(\neg M \wedge \neg A)$  must be False. Therefore we cannot make all the premises true when the conclusion is False UNDER THIS ASSIGNMENT. Does this make the argument valid? NO. We must show that it is impossible to make the premises True while the conclusion is False under ANY truth value assignment. So we must test the other possibilities. Suppose we try case (i) where  $M$  is True and  $U$  is False. Here the first premise cannot be True.

So let's try case (ii) where  $M$  is True and  $U$  is True. Since  $U$  is True then the first premise is True. Since  $M$  is True then  $\neg M$  is False, so the conclusion of the second premise is False. Thus the only way the second premise could be True is if  $A$  is False. So making  $A$  False makes the second premise True. Now in the third premise, since  $\neg M$  is False,  $(\neg M \wedge \neg A)$  is False and thus,  $\neg(\neg M \wedge \neg A)$  must be True. Therefore we have made all three premises True while the conclusion is False, so the argument is INVALID.

15) Note that the conclusion is an "and" statement.

The trees are tall **and** the land is rich.

When is an "and" statement False?

What does this mean in terms of showing whether the argument is Valid or Invalid?

$$\begin{array}{l}
 17) \quad N \rightarrow \neg(W \wedge \neg S) \quad \text{Invalid} \\
 \neg(W \wedge \neg S) \vee (\neg I \rightarrow C) \\
 I \wedge N \\
 \hline
 \neg C
 \end{array}$$